naturally depend on the quality of the field measurements.

Conclusions

It is the most significant point at issue in those discussions whether the parameter λ should have the rationality of soil mechanics. In the case of an unspecified slope, because λ has such rationality it does not neccessarily follow that the prediction method is useful. However, all discussers require it and we can get good suggestions for them. We slould also modify the mechanism of failure and the model of analysis in the presented paper, if possible.

We reconsider that λ is too essential parameter to predict a slope failure. Therefore, the most desirable mechanism which should be incorporated in this analysis is the model presented by Mostyn which predicts the effective stress in the slope during rainfall. We will propose the new model to estimate the coefficient of permeability instead of λ because it is the most uncertain and basic parameter in unsaturated seepage analysis. This direction can be found by these useful discussions.

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A STUDY ON LATERAL DASHPOTS FOR SOIL-STRUC-TURE INTERACTION AND ITS APPLICATION TO A SIMPLIFIED TECHNIQUE[i]

Discussion by GEORGE GAZETAS[ii]

Introduction

This interesting paper addresses the timely question of developing suitable simplified methods for computing the seismic response of realistic three-dimensional soil-foundation systems. The complexity of the problem studied in the paper is indeed formidable: rectangular foundation plan, two adjacent foundations, different depths of embedment, excitation consisting of inclined waves, soil modeled as homogeneous halfspace or stratum. Hence, their attempts to develop a semi-rigorous boundary-element formulation and, furthermore, based on the results of such a formulation, to develop an engineering simpli-fied method "using soil columns with dashpots", is truly commendable and justifies the publi-shing of their paper.

In this discussion the writer will try to pro-vide a critical perspective on the nature of the developed simplified procedure, without for a moment intending to negate the significant overall contribution of the authors.

Radiation damping paradox

One of the main theses of the authors' effort to develop a simplified method is based on the presumption that the radiation dashpot con-stants in a 3-D environment (such as under a square or a circular foundation) are greater than the dashpot constants in a 2-D environ-ment (such as under strip or very elongated foundations). Hence 2-D slices of the soil-foundation system could be used to obtain the 3-D response, if only proper viscous dashpots were attached onto the faces the soil slices (Figs. 1, 10, 11 of the paper).

Unfortunately, however, this intuitive pre-sumption is a fallacy. For, as it will be shown in the sequel, this unfounded hypothesis con-tradicts reality for all modes of vibration, on homogeneous, inhomogeneous, and layered soil profiles, for shallow foundations.

Before providing the evidence, a historical background is noteworthy. Admittedly, at first glance the aforementioned presumption seems to be a reasonable hypothesis supported
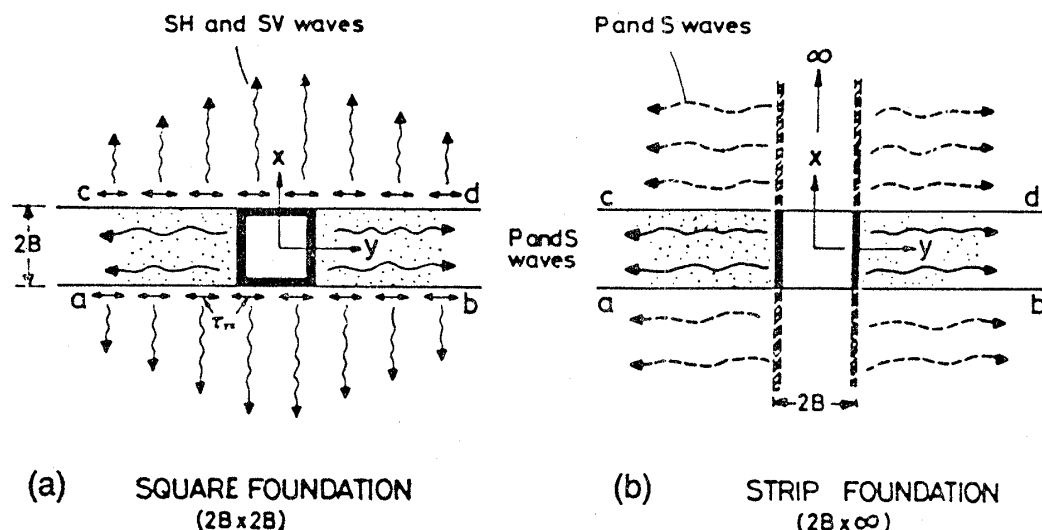
Fig. 14. Square versus strip foundation. (a) Square: shear stresses τ_{yx} and τ_{xz} could in principle develop on ab and cd, generating SH and SV waves, and thereby radiating additional energy away from the foundation. (b) Strip: the slice of soil between ab and cd carries P and S waves parallel to y, and no dynamic stresses can be created on ab and cd. Based on this "observation" it has been arqued (erroneously) that square foundations generate more damping (per unit area) than strips. Reality does not support this (apparently logical) conclusion (: paradox)

by intuition: it is the analogous of $2+1=3$ ($2-D$ slice+$1-D$ face dashpots=$3-D$ problem)! The argument in favor of this presumption has been that in a $3-D$ environment waves originating from the foundation could, in principle, spread outward and downward in all three directions; hence, they could carry away larger amounts of energy (per unit foundation contact area) than in a $2-D$ environment, where the emitted waves can spread in two directions only. This "common sense" (but erroneous) argument is illustrated in Fig. 14.

In the mid–1970's this "common–sense" presumption motivated some attempts to model $3-D$ soil-structure interaction geometries using $2-D$ plane-strain finite elements but with special viscous dashpots attached on their lateral faces to "absorb" the SH and SV waves that would have spread in the suppressed third direction. It will become evident in the sequel that the addition of such dashpots would aggravate the problem (i. e., the discrepancy between $2-D$ and $3-D$ solutions) by generating artificially high damping.

In reality, $3-D$ surface foundations (squares, circles) generate consistently, and for low frequencies considerably, smaller amount of radiation damping per unit contact area (or area-moment of inertia) than $2-D$ surface foundations (very long rectangles, strip footings). It seems that in wave propagation phenomena two plus one does not necessarily equal three. In fact, in the case of foundations resting on almost any soil deposit, two plus one is even less than two !!, as the addition of more surface available for wave transmission results in less not more damping.

Here is some evidence of this paradox:
1. *On a homogeneous halfspace.*—Fig. 15 portrays a comparison of the radiation data associated with strip (L/B=infinity), circular or square (L/B=1), and rectangular foundations of various aspect ratios L/B, all resting on a homogeneous halfspace with S-wave velocity V_s. Specifically, Fig. 15 (a) plots the variation with frequency of the normalized radiation damping coefficient, $C_h/\rho V_{La}A$, for *horizontal* (lateral) motion. A=the area of the foundation-soil contact surface. (For the strip, both C_h and $A=2B$ are defined per unit length of footing.) Fig. 15 (b) plots the normalized *vertical* radiation dashpot coeffi-
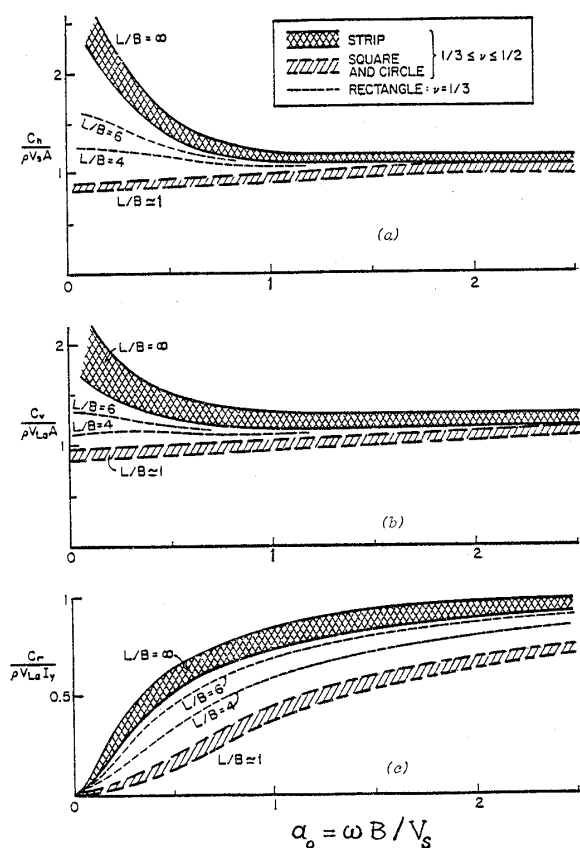
Fig. 15. Influence of foundation shape and excitation frequency on the horizontal, vertical and rocking radiation damping coefficients of surface foundations on a homogeneous halfspace; the strip (L/B ⌣∞) has invariably larger damping than the circular and square (L/B≃1) foundations. (Results are from rigorous solutions.)

cient, $C_v/\rho V_{La}A$, versus a_0. The "apparent" propagation ("Lysmer's analog") velocity V_{La} (adopted by the authors and denoted as V_L) has been introduced and advocated in a number of publications by Gazetas & Dobry, and it is based on Lysmer's pioneering study on surface circular foundations. Finally, in Fig. 15 (c), the rocking radiation damping coefficient, C_r, has been normalized by dividing by $\rho V_{La}I_y$, where I_y= the contact area moment of inertia about the y axis. It is noted that the numerical/analytical results used to compile this figure have been published by Karasudhi, Luco, Veletsos, Gazetas, Wong, Domiuguez, Roesset, Rucker, and others, and are well known to the profession.

The following conclusions are drawn from Fig. 15:

● Of all the foundation shapes considered, and for a given frequency, it is indeed the strip which produces the largest amount of radiation damping per unit contact area (or per unit contact area moment of inertia, in the case of rocking). By contrast, the circular and square shapes are associated with the lowest values of radiation damping, in all three modes of vibration.

● In the low frequency range, $0 < a_0 < 0.50$, radiation damping is particularly sensitive to variations in foundation shape, and the differences between plane-strain (strip) and 3-D (square or circle) geometries are very significant. For example, at the middle of this range, i. e. $a_0 \approx 0.25$, the dashpot coefficients of the strip footing are larger than those of the square footing by the following approximate amounts: 100% for the horizontal motion, 60% for the vertical motion, and 300% for the rocking motion.

● At high frequencies all normalized radiation damping coefficients, $C_h/\rho V_sA$, $C_v/\rho V_{La}A$ and $C_r/\rho V_{La}I_y$, tend to become equal to about 1, regardless of foundation shape. The phenomenon is analogous to the high-frequency behavior of the acoustic impedance of a loudspeaker. Consequently, the differences in damping between strip and square foundations tend to decrease at high frequencies. The normalized C_h and C_v attain practically identical values for all foundation shapes for $a_0 > 1.5$, while for rocking $C_r/\rho V_{La}I_y$ becomes practically independent of shape for $a_0 > 3$.

● Rectangular foundations with aspect ratio, L/B, between 1 (square) and ∞ (strip), develop a radiation damping of intermediate size compared with these two extreme cases. Moreover, as L/B increases and hence the footing geometry gets closer to the 2-D geometry of the strip, radiation damping invariably increases.

In conclusion, on a homogeneous halfspace radiation damping is consistently and appreciably larger in plane-strain (2-D) than in truly 3-D loading environments. Only at high
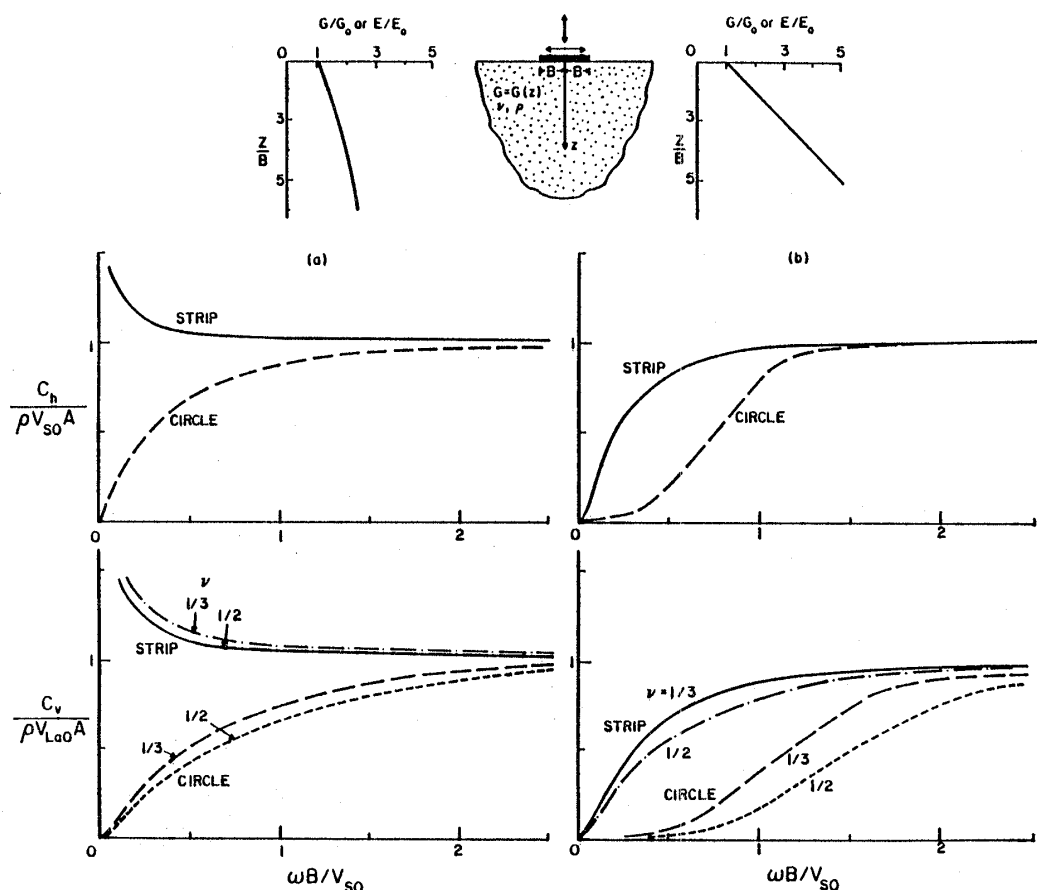
Fig. 16. Horizontal and vertical radiation damping of strip and circular foundations on two inhomogeneous halfspaces: (a) soil moduli increasing parabolically with depth; (b) soil moduli increasing linearly with depth. Again the strip has larger damping than the circle

frequencies do the plane-strain and 3–D values become approximately equal; but *in no case* is the plane-strain damping smaller. Consequently, the addition of viscous dashpots at the lateral faces, as advocated in the paper, would *worsen rather than improve* the situation.

2. *Inhomogeneous and layered soil deposits.* —Figs. 16 and 17 compare the radiation damping coefficients of a circular (radius $2\,B$) and a strip ($2\,B$ by infinity) foundation resting on the surface of four realistically-idealized soil profiles: The first two profiles have a shear modulus increasing continuously with depth, in the form

$$G=G_0(1+z/B)^{1/2} \text{ and } G=G_0(1+z/B)$$

The other two profiles are homogeneous layers underlain by a rigid base, and having thicknesses $H=4\,B$ and $H=8\,B$, respectively. The results plotted in these two figures are based

on the work of Kausel, Roesset, Luco, Gazetas & Dobry, and others.

The main conclusion drawn previously for a homogeneous halfspace from Fig. 15 is also valid (essentially with no exception) for the four non-homogeneous deposits of Figs 16 and 17: 2–D environments are associated with more, not less, damping per unit contact plan area or moment of inertial than truly 3–D environments. An attempt for an explanation the deeper causes of this "paradoxical" behavior has been given by Gazetas (1987). But note that other researchers had pointed out this fact much earlier (Luco & Hadjian 1974).

The results of the authors

It is thus expected (if what the writer have presented is correct) that by attaching dashpots on the faces of the soil columns under the
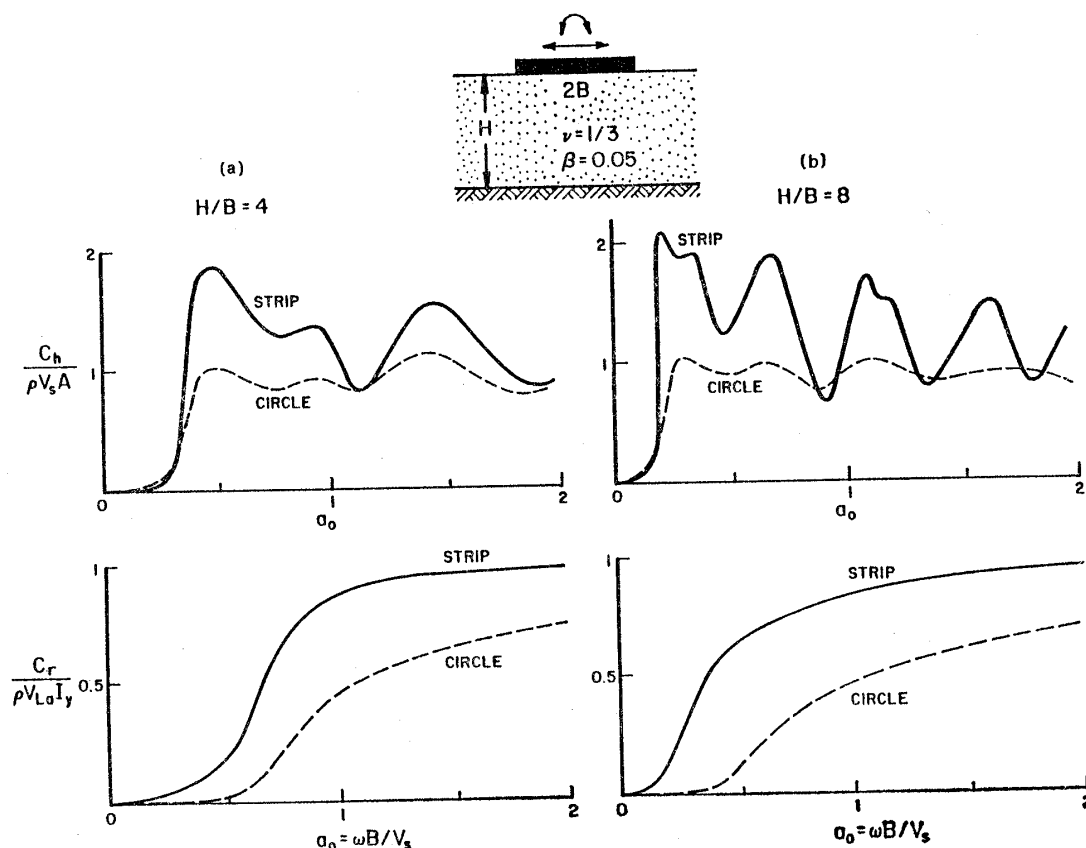
Fig. 17.   Horizontal and rocking radiation damping of strip and circular foundations
on soil layer over rigid rock:
(a) soil layer thickness $H=4B$
(b) soil layer thickness $H=8B$. The largest damping is again associated with the
strip

foundation base, the authors worsen rather than improve the discrepancy with the 3-D reality they are trying to simulate.   Here is some examples:

1.   Refer to Figures 10–11 and the associated Table 1 of the discussed paper, and consider a square footing ($2B$ by $2B$) resting on the surface of a homogeneous halfspace and subjected to vertical oscillations.   The impedance is expressed in the form

$$K_Z = \overline{K}_Z + i\,\omega\,C_Z \qquad (33)$$

where the correct (3-D) dashpot constant (Lysmer & Richart 1966, Veletsos & Wei 1970) is approximately independent of frequency and equals (within 10%):

$$C_Z = \rho V_L A, \text{ where } A = 4B^2 \qquad (34)$$

Now let us examine the proposed solution: The governing equation of motion for the soil column with the added dashpots on the sides

is given in the second row and second column of Table 1 of the paper.   The solution (based on this equation) for the impedance function, given in the third column, is:

$$\text{Impedance} = i\,\omega\,\rho V_L A\left(1 - \frac{i\,\eta}{\rho\omega}\right) \qquad (35)$$

where, for a square, $B=D$ and according to the last column of Table 1, i. e. for the 3-D case,

$$\eta = \eta(3\text{-}D) = 2\frac{\rho V_s}{B} \qquad (36)$$

Substituting this last expression into Eq. 35 leads to:

$$\text{Impedance} = i\,\omega\rho\,V_L A\sqrt{1 - i\frac{2\,\rho V_s}{B}/\rho\omega}$$
$$= i\,\omega\rho\,V_L A\,\sqrt{1 - 2\,i/a_0} \qquad (37)$$

where $a_0 = \omega\,B/V_s$.   Denoting $\phi = \arctan(2/a_0)$, Eq. (37) becomes

$$\text{Impedance} = i\,\omega\rho\,V_L A\left(1 + \frac{4}{a_0^2}\right)^{1/4} \cdot e^{-i\phi/2}$$

$$= \omega \rho V_L A (1+4/a_0^2)^{1/4} [\sin(\phi/2) + i \cos(\phi/2)] \tag{38}$$

from which the authors' dashpot constant is recovered:

$$C_{\text{AUTHORS'}} = \rho V_L A (1+4/a_0^2)^{1/4} \cos(\phi/2) \tag{39}$$

Compare now Eq. 39 with the essentially exact dashpot value of Eq. 34. The ratio

$$\frac{C_{\text{AUTHORS'}}}{C_{\text{CORRECT}}} = \left(1+\frac{4}{a_0^2}\right)^{1/4} \cos(\phi/2) \tag{40}$$

is greater than 1 for all frequencies. In fact, as depicted in Table 4 herein, at low frequencies this ratio is far greater than 1, but it does tend to unity (asymptotically) at high frequencies.

Table 4. Evaluation of the authors' simplified method for vertical damping on a halfspace

a_0	0.1	0.2	0.5	1.0	1.5	2.0	3.0
$\dfrac{C_{\text{AUTHORS'}}}{C_{\text{CORRECT}}}$	3.2	2.3	1.6	1.5	1.3	1.2	1.04

2. Refer to Figs. 3 a, 3 b, 3 c, 7 a, and 7 b of the discussed paper. Although it is a little hard to distinguish in some cases, in all of these plots:

● the imaginary part $(=\omega C)$ of the 2-d impedances is much closer to the true 3-D than to the Approx. 3-D (i. e., to the 2-D slice plus face dashpots) imaginary part; hence, adding the face dashpots did not help

● Moreover, in the (practically-significant) low frequency range, the Approx. 3-D imaginary part lies on the wrong side of the 2-D solution (see for instance Fig. 3 b); i. e. while the true 3-D curve lies below the 2-D solution (in accordance with the "paradox"), the Approx. 3-D overpredicts the 2-D curve— apparently as a result of the spurious additional damping contributed by the face dashpots to the already-large 2-D damping. (Fig. 18 herein is a blow-up of the author's Fig. 3 b showing the imaginary part of the rocking impedance, to make the above points easier to see.)

Some final comments

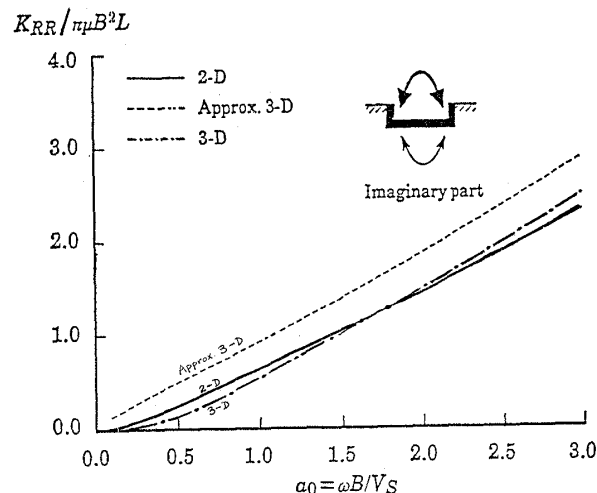In all fairness to the authors, however, it is

$K_{RR}/\pi\mu B^2 L$

Fig. 18. The authors' Fig. 3 b for ωC (imaginary part of the impedance) in rocking. It is clear that the 2-D solution (without any "correction") is much better than the Approx. 3-D one. Moreover, for the low frequency range $a_0 < 1$ (which is of greatest interest in earthquake engineering), where even the 2-D solution overpredicts the very small damping of the 3-D reality, the addition of face-dashpots in the Approx. 3-D formulation worsens the situation substantially: the dashpot constant is overpredicted by an (unsafe) factor of up to 4

emphasized that the "paradoxical picture" outlined so far is strictly applicable to damping generated at the base of single foundations. Two other factors that were considered in the paper, namely, *embedment* and the presence of an adjacent foundation further complicate the pattern of radiation of the wave energy, and may in some cases reduce considerably the adverse consequences of the added dashpots, thereby rendering their results acceptable within the realm of an engineering approximation.

It is also true that, while the dashpot constant C is invariably overpredicted with the authors' approximation, the dynamic stiffness, $\overline{K}$, may in certain cases be underpredicted with a 2-D approximation; hence the equivalent damping ratio of a massless foundation, $\omega C / 2\overline{K}$, may only moderately be affected. (However, for the practically-interesting case of a

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mass–possessing structure, even such a cancellation of errors would hardly take place.)

Concluding, the writer would like to stress again that the authors must be praised for both: (i) attempting to solve such a difficult realistic engineering problem, and (ii) focusing on the development of simplified solution for this class of soil–structure interaction problems.

They should be encouraged to continue their work along similar lines taking the foregoing comments into account and resorting to sound wave–propagation approximations. Deleting the "face" dashpots altogether would be one (easy) first step in the right direction.

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A STUDY ON LATERAL DASHPOTS FOR SOIL–STRUCTURE INTERACTION AND ITS APPLICATION TO A SIMPLIFIED TECHNIQUE[i]

Closure by Nobuo Fukuwa[ii]
and Shoichi Nakai[iii]

Introduction

The writers wish to express their sincere gratitude for the discussion on our paper by the discusser, Prof. G. Gazetas, who has been widely studying soil–structure interaction problems from the practical view point. However the writers want to point out that his discussion seems to come from a fatal misunderstanding on our paper, which would be immediately resolved if the discusser carefully reads the introduction of our paper.

In the first part of our paper, we examined the effect of viscous dashpots on the soil-structure interaction using the boundary element method. Through this examination, the applicability criteria of the approximate three-dimensional analysis was proposed. Since this type of analysis was found effective for a foundation embedded in a half–space, we extended the idea of viscous dashpots and proposed the simplified analysis technique based on a one-dimensional soil column.

Radiation damping between 2–D vs 3–D

The main point of the discussion is:

(this paper is) based on the presumption that the radiation dashpot constant in a 3–*D* environment are greater than the dashpot constants in a 2–*D* environment.

However, the writers have not made this pre-